Z notation

This file is a translation of the German original which can be found at `http://www.pst.informatik.uni-muenchen.de/lehre/WS0203/gse/z-notation`. The following tables provide an overview of the notation used in Z. The LaTeXinput format is accepted by most Z tools, e.g. Z/Eves or Jaza. However, you need the file `zed-csp.sty` or `z-eves.sty`, respectively to compile your latex source file. All inputs must be declared inside a `zed` environment, i.e. between `\begin{zed}` and `\end{zed}`.

declarations and definitions		
LaTeX input	output	explanation
`[type]`	$[type]$	basic type
`id~params == expr`	$id\ params == expr$	shortcut
`T ::= c |` `d \ldata U \rdata`	$T ::= c \mid d\langle\langle U\rangle\rangle$	definition of free types

`\begin{axdef}`
`x: T`
`\where`
`P`
`\end{axdef}`
	$x : T$
	———
	P
		axiomatic definitions

`\begin{gendef}[X]`
`x: X`
`\where`
`P`
`\end{gendef}`
	$[X]$
	$x : X$
	———
	P
		generic definitions

`\begin{schema}{S}`
`x: T`
`\where`
`P`
`\end{schema}`
	S
	$x : T$
	———
	P
		schema definition

`S \defs [x:T | P]`	$S \widehat{=} [x : T \mid P]$	schema definition in "horizontal" notation

schema expressions		
LaTeX input	output	explanation
`Id', Id!, Id?, Id_n`	$Id', Id!, Id?, Id_n$	decorations (n digit)
`\Delta S`	ΔS	shortcut for $S \land S'$
`\Xi S`	ΞS	shortcut for $S \land S' \land \theta S = \theta S'$
`\lnot S, S \land T,` `S \lor T`	$\neg S,\ S \land T,$ $S \lor T$	logical schema conjunctions
`\forall x:T @ S` `\exists x:T @ S`	$\forall x : T \bullet S$ $\exists x : T \bullet S$	quantifiers (also possible: $\exists_1$)
`S \hide (x)`	$S \setminus (x)$	Hiding, same as $\exists x : T \bullet S$, where T is the type of x in S
`S \project T`	$S \upharpoonright T$	projection, same as $(S \land T) \setminus (\vec{x})$, where $\vec{x}$ represents all variables in S which are *not* in T
`S \semi T`	$S \fatsemi T$	sequential composition
`S \pipe T`	$S \gg T$	pipe-composition
`\pre S`	pre S	precondition (hiding of output and striked out variables, respectively)

<table>
<tr><th colspan="3">expressions</th></tr>
<tr><th>LaTeX input</th><th>output</th><th>explanation</th></tr>
<tr><td><code>\{ x:T | P @ E \}</code></td><td>$\{x : T \mid P \bullet E\}$</td><td>set comprehension (E optional), e.g.
$\{x : \mathbb{Z} \mid x > 5 \bullet x * x\} = \{36, 49, 64, \ldots\}$</td></tr>
<tr><td><code>(\lambda x:T | P @ E)</code></td><td>$(\lambda x : T \mid P \bullet E)$</td><td>$\lambda$-expression, same as $\{x : T \mid P \bullet (x, E)\}$</td></tr>
<tr><td><code>f~x</code></td><td>$f\ x$</td><td>function application</td></tr>
<tr><td><code>(\mu x:T | P @ E)</code></td><td>$(\mu x : T \mid P \bullet E)$</td><td>unique specification (E optional), e.g.
$(\mu x : \mathbb{N}_1 \mid x * x = x \bullet x + x) = 2$</td></tr>
<tr><td><code>\IF P \THEN E1 \ELSE E2</code></td><td>if P then $E1$ else $E2$</td><td>conditional expression</td></tr>
<tr><td><code>\LET x==E1 @ E2</code></td><td>let $x == E1 \bullet E2$</td><td>local definition</td></tr>
</table>

formulae		
LaTeX input	output	explanation
`x = y, x \neq y`	$x = y,\ x \neq y$	(in-)equality
`x \in S, x \notin S`	$x \in S,\ x \notin S$	(non-)membership
`S \subseteq T`	$S \subseteq T$	subset
`S \subset T`	$S \subset T$	proper subset
`\lnot P`	$\neg P$	negation
`P \land Q`	$P \land Q$	conjunction
`P \lor Q`	$P \lor Q$	disjunction
`P \implies Q`	$P \Rightarrow Q$	implication
`P \iff Q`	$P \Leftrightarrow Q$	equivalence
`\forall x:T @ P`	$\forall x : T \bullet P$	universal quantifier
`\exists x:T @ P`	$\exists x : T \bullet P$	existential quantifier
`\exists_1 x:T @ P`	$\exists_1 x : T \bullet P$	unique existence
`a \inrel{R} b`	$a\ \underline{R}\ b$	infix-relation, same as $(a, b) \in R$

sets		
LaTeX input	output	explanation
`\emptyset`	$\emptyset$	empty set
`\{a, b\}`	$\{a, b\}$	enumeration of finite sets
`S \cup T, S \cap T`	$S \cup T,\ S \cap T$	union, intersection
`S \setminus T`	$S \setminus T$	complement
`\power S, \power_1 S`	$\mathbb{P} S,\ \mathbb{P}_1 S$	power set ($\mathbb{P}_1 S = \mathbb{P} S \setminus \emptyset$)
`S \cross T`	$S \times T$	cartesian product
`\bigcup TT`	$\bigcup TT$	generalized union, $a \in \bigcup TT \Leftrightarrow \exists S \in TT \bullet a \in S$
`\bigcap TT`	$\bigcap TT$	generalized intersection, $a \in \bigcap TT \Leftrightarrow \forall S \in TT \bullet a \in S$

pairs and tuples		
LaTeX input	output	explanation
`(a, b)`	(a, b)	pair and tuple construction (arbitrary arity)
`a \mapsto b`	$a \mapsto b$	ordered pair („maplet“), same as (a, b)
`t.n`	$t.n$	selection of the nth component, e.g. $(a, b, c).2 = b$
`first~p`	$\mathit{first}\, p$	first component, $\mathit{first}(a, b) = a$ (same as $p.1$)
`second~p`	$\mathit{second}\, p$	second component, $\mathit{second}(a, b) = b$ (same as $p.2$)

relations		
LaTeX input	output	explanation
`X \rel Y`	$X \leftrightarrow Y$	set of relations, $X \leftrightarrow Y = \mathbb{P}(X \times Y)$
`\dom R`	$\operatorname{dom} R$	domain, $\operatorname{dom} R = \{ x : X;\ y : Y \mid x\, \underline{R}\, y \bullet x \}$
`\ran R`	$\operatorname{ran} R$	range, $\operatorname{ran} R = \{ x : X;\ y : Y \mid x\, \underline{R}\, y \bullet y \}$
`\id X`	$\operatorname{id} X$	identity relation, $\operatorname{id} X = (\lambda x : X \bullet x)$
`Q \comp R`	$Q \fatsemi R$	composition of relations, $Q \fatsemi R = \{ x : X;\ z : Z \mid (\exists y : Y \bullet x\, \underline{Q}\, y \wedge y\, \underline{R}\, z) \bullet x \mapsto z \}$
`R \circ Q`	$R \circ Q$	backward composition, $R \circ Q = Q \fatsemi R$, $(f \circ g)\, x = f(g\, x)$
`R \inv`	$R^{\sim}$	inverse relation, $R^{\sim} = \{ x : X;\ y : Y \mid x\, \underline{R}\, y \bullet y \mapsto x \}$
`A \dres R`	$A \lhd R$	domain restriction, $A \lhd R = \{ x : X;\ y : Y \mid x\, \underline{R}\, y \wedge x \in A \bullet x \mapsto y \}$
`A \ndres R`	$A \ndres R$	domain anti-restriction, $A \ndres R = \{ x : X;\ y : Y \mid x\, \underline{R}\, y \wedge x \notin A \bullet x \mapsto y \}$
`A \rres R`	$A \rhd R$	range restriction, $A \rhd R = \{ x : X;\ y : Y \mid x\, \underline{R}\, y \wedge y \in B \bullet x \mapsto y \}$
`A \nrres R`	$A \nrres R$	range anti-restriction, $A \nrres R = \{ x : X;\ y : Y \mid x\, \underline{R}\, y \wedge y \notin B \bullet x \mapsto y \}$
`R \limg A \rimg`	$R \llparenthesis A \rrparenthesis$	relational mapping, $R \llparenthesis A \rrparenthesis = \operatorname{ran}(A \lhd R)$
`Q \oplus R`	$Q \oplus R$	overwriting, $Q \oplus R = (\operatorname{dom} R \ndres Q) \cup R$
`R \plus`	R^{+}	transitive closure, $R^{+} = \bigcap \{ Q : X \leftrightarrow X \mid R \subseteq Q \wedge Q \fatsemi Q \subseteq Q \}$
`R \star`	R^{*}	reflexive transitive closure, $R^{*} = \operatorname{id} X \cup R^{+}$
`R^{k}`	R^{k}	relational iteration, $R^{0} = \operatorname{id} X$, $R^{k+1} = R \fatsemi R^{k}$, $R^{-k} = (R^{\sim})^{k}$

functions		
LaTeX input	output	explanation
`X \pfun Y`	$X \pfun Y$	partial function, $\{ R : X \leftrightarrow Y \mid \forall x : X;\ y_1, y_2 : Y \bullet x\, \underline{R}\, y_1 \wedge x\, \underline{R}\, y_2 \Rightarrow y_1 = y_2 \}$
`X \fun Y`	$X \rightarrow Y$	total function, $X \rightarrow Y = \{ f : X \pfun Y \mid \operatorname{dom} f = X \}$
`X \pinj Y`	$X \pinj Y$	partial injection, $\{ f : X \pfun Y \mid \forall x_1, x_2 : \operatorname{dom} f \bullet f\, x_1 = f\, x_2 \Rightarrow x_1 = x_2 \}$
`X \inj Y`	$X \rightarrowtail Y$	total injection, $X \rightarrowtail Y = (X \rightarrow Y) \cap (X \pinj Y)$
`X \psurj Y`	$X \psurj Y$	partial surjection, $X \psurj Y = \{ f : X \pfun Y \mid \operatorname{ran} f = B \}$
`X \surj Y`	$X \twoheadrightarrow Y$	total surjection, $X \twoheadrightarrow Y = (X \rightarrow Y) \cap (X \psurj Y)$
`X \bij Y`	$X \bij Y$	bijection, $X \bij Y = (X \rightarrowtail Y) \cap (X \twoheadrightarrow Y)$

numbers		
LaTeX input	output	explanation
\num	$\mathbb{Z}$	integers
\nat	$\mathbb{N}$	natural numbers, $\mathbb{N} = \{ n : \mathbb{Z} \mid n \geq 0 \}$
\nat_1	$\mathbb{N}_1$	positive natural numbers, $\mathbb{N}_1 = \mathbb{N} \setminus \{0\}$
+, -, *	$+, -, *$	arithmetic operations ($\mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$)
\div, \mod	div, mod	division, modulo ($\mathbb{Z} \times (\mathbb{Z} \setminus \{0\}) \rightarrow \mathbb{Z}$)
<, \leq, >, \geq	$<, \leq, >, \geq$	arithmetic comparisons ($\mathbb{Z} \leftrightarrow \mathbb{Z}$)
succ	*succ*	successor ($\mathbb{N} \rightarrow \mathbb{N}$)
a \upto b	$a \mathrel{..} b$	intervals, $a \mathrel{..} b = \{ n : \mathbb{Z} \mid a \leq n \land n \leq b \}$
min~S, max~S	$min\, S$, $max\, S$	minimum/maximum element of a set of number (if ex.) $min\, S = (\mu\, m : S \mid (\forall n : S \bullet m \leq n))$

finite sets		
LaTeX input	output	explanation
\finset S	$\mathbb{F}\, S$	set of the finite subsets of S, $\mathbb{F}\, S = \{ s : \mathbb{P}\, S \mid \exists n : \mathbb{N} \bullet \exists f : 1 \mathrel{..} n \rightarrow S \bullet \operatorname{ran} f = S \}$
\finset_1 S	$\mathbb{F}_1\, S$	non-empty finite subsets, $\mathbb{F}_1\, S = \mathbb{F}\, S \setminus \emptyset$
\# S	$\#S$	cardinality of a finite set, $\#S = (\mu\, n : \mathbb{N} \mid (\exists f : 1 \mathrel{..} n \rightarrowtail S \bullet \operatorname{ran} f = S))$
X \ffun Y	$X \mathrel{+\!\!\!\!\!\!\rightarrow\!\!\!\!\!\!+} Y$	finite partial function, $X \mathrel{+\!\!\!\!\!\!\rightarrow\!\!\!\!\!\!+} Y = \{ f : X \mathrel{+\!\!\!\!\!\!\rightarrow} Y \mid \operatorname{dom} f \in \mathbb{F}\, X \}$
X \finj Y	$X \mathrel{\rightarrowtail\!\!\!\!\!\!+\!\!\!\!+} Y$	finite partial injection, $X \mathrel{\rightarrowtail\!\!\!\!\!\!+\!\!\!\!+} Y = (X \mathrel{+\!\!\!\!\!\!\rightarrow\!\!\!\!\!\!+} Y) \cap (X \mathrel{\rightarrowtail\!\!\!\!\!\!+} Y)$

multisets		
LaTeX input	output	explanation
\bag S	$\operatorname{bag} S$	multiset over S, $\operatorname{bag} S = (S \mathrel{+\!\!\!\!\!\!\rightarrow} \mathbb{N}_1)$
\lbag a, a, b\rbag	$[\![a, a, b]\!]$	enumeration of finite subsets, same as $\{a \mapsto 2, b \mapsto 1\}$
B \bcount x	$B \sharp x$	frequency of x in B, $B \sharp x = (\lambda\, x : S \bullet 0) \oplus B$
n \otimes B	$n \otimes B$	scaling, $n \otimes B = (\lambda\, x : \operatorname{dom} B \bullet x \mapsto n * (B\, x))$
x \inbag B	$x \text{ in } B$	membership in a multiset, $x \text{ in } B \Leftrightarrow x \in \operatorname{dom} B$
A \subbageq B	$A \sqsubseteq B$	sub-multiset relation, $A \sqsubseteq B \Leftrightarrow \forall x : S \bullet A \sharp x \leq B \sharp x$
A \uplus B	$A \uplus B$	multiset union, $(A \uplus B) \sharp x = A \sharp x + B \sharp x$
A \uminus B	$A \cup\!\!\!\!\!- B$	multiset complement, $(A \cup\!\!\!\!\!- B) \sharp x = max\, \{0, A \sharp x - B \sharp x\}$
items~s	*items s*	multiset of the elements of a sequence, $(items\, s) \sharp x = \#(s \upharpoonright \{x\})$

finite sequences		
LaTeX input	output	explanation
`\seq S`	$\mathrm{seq}\, S$	set of the finite sequences of S, $\mathrm{seq}\, S = \{ s : \mathbb{N} \mathrel{⇻} S \mid \exists n : \mathbb{N} \bullet \mathrm{dom}\, s = 1 \mathinner{..} n \}$
`\seq_1 S`	$\mathrm{seq}_1\, S$	non-empty sequence, $\mathrm{seq}_1\, S = \{ s : \mathrm{seq}\, S \mid \# s > 0 \}$
`\iseq S`	$\mathrm{iseq}\, S$	duplicate-free sequence, $\mathrm{iseq}\, S = (\mathrm{seq}\, S) \cap (\mathbb{N} \mathrel{⤔} S)$
`\langle a, a, b \rangle`	$\langle a, a, b \rangle$	enumeration of a finite sequence, same as $\{ 1 \mapsto a, 2 \mapsto a, 3 \mapsto b \}$
`s \cat t`	$s \frown t$	concatenation, $s \frown t = s \cup \{ n : \mathrm{dom}\, t \bullet (n + \# s) \mapsto t(n) \}$
`rev~s`	$\mathit{rev}\, s$	inversion, $\mathit{rev}\, s = (\lambda n : \mathrm{dom}\, s \bullet s(\# s - n + 1))$
`head~s`	$\mathit{head}\, s$	first element, $\mathit{head}\, s = s(1)$
`tail~s`	$\mathit{tail}\, s$	remainder of the sequence, $\mathit{tail}\, s = (\lambda n : 1 \mathinner{..} \# s - 1 \bullet s(n+1))$
`last~s`	$\mathit{last}\, s$	last element, $\mathit{last}\, s = s(\# s)$
`front~s`	$\mathit{front}\, s$	sequence without the last element, $\mathit{front}\, s = (1 \mathinner{..} \# s - 1) \lhd s$
`squash~f`	$\mathit{squash}\, f$	compactification, $(\mu g : 1 \mathinner{..} \# f \mathrel{⤖} \mathrm{dom} f \mid g^\sim \mathbin{⨾} \mathit{succ} \mathbin{⨾} g \subseteq (\_ < \_)) \mathbin{⨾} f$
`A \extract s`	$A \upharpoonleft s$	extraction of the elemente using indexes in A, $A \upharpoonleft s = \mathit{squash}\,(A \lhd s)$
`s \filter A`	$s \upharpoonright A$	subsequence of the elements of s which are contained in A, $s \upharpoonright A = \mathit{squash}\,(s \rhd A)$
`s \prefix t`	$s \mathrel{\textsf{prefix}} t$	prefix relation, $s \mathrel{\textsf{prefix}} t \Leftrightarrow \exists v : \mathrm{seq}\, S \bullet s \frown v = t$
`s \suffix t`	$s \mathrel{\textsf{suffix}} t$	suffix relation, $s \mathrel{\textsf{suffix}} t \Leftrightarrow \exists v : \mathrm{seq}\, S \bullet v \frown s = t$
`s \inseq t`	$s \mathrel{\textsf{in}} t$	subsequence, $s \mathrel{\textsf{in}} t \Leftrightarrow \exists u, v : \mathrm{seq}\, S \bullet u \frown s \frown v = t$
`\dcat s`	$\frown / s$	concatenation of all sequences in s, $\frown / \langle \rangle = \langle \rangle, \quad \# s > 1 \Rightarrow \frown / s = (\mathit{head}\, s) \frown (\frown / \mathit{tail}\, s)$